# Introduction to Nonlinear Dynamics: Midterm exam
# September 16, 2005

Attempt any five questions. Each question is worth 20 points.

1. Which of the following dynamical systems on $\mathbb{R}$ has a sink at 0? Justify your answer.
   (a) $\dot{x} = -7x^{14}$
   (b) $\dot{x} = -14x^7$

2. Consider the dynamical system $\dot{U} = AU$, where $A$ is the matrix

$$\begin{pmatrix} -1 & 0 \\ \epsilon & -1 \end{pmatrix}$$

   Let $U = (x, y)^t$. Show that $F(x, y) = x^2 + y^2$ is a Liapunov function if $|\epsilon| < 2$.

3. Show that the dynamical system $\dot{x} = x^2 - 4x + 4$ on $\mathbb{R}$ is not locally structurally stable.

4. For a hyperbolic dynamical system $\dot{U} = AU$ on $\mathbb{R}^n$ (i.e., with $A$ non-singular and with $Re(\lambda) \neq 0$ for every eigenvalue $\lambda$ of $A$), let $\sigma(A)$ be the number of eigenvalues $\lambda$ of $A$ such that $Re(\lambda) < 0$ (counted with multiplicity). Show that if $\dot{U} = AU$ and $\dot{U} = BU$ are dynamical systems on $\mathbb{R}^n$ with $\sigma(A) = \sigma(B)$, then the systems are topologically conjugate.

5. Show that two linear dynamical systems $\dot{U} = AU$ and $\dot{U} = BU$ on $\mathbb{R}^n$, with $A$ and $B$ non-singular, are smoothly conjugate if and only if $A$ and $B$ are conjugate linear transformations (i.e., there is an invertible linear map $C : \mathbb{R}^n \to \mathbb{R}^n$ with $AC = CB$).

6. Consider a smooth 1-dimensional dynamical system $\varphi_t : \mathbb{R} \to \mathbb{R}$, $t \in \mathbb{R}$, with no equilibrium points. Show that there is only one orbit, i.e., for $x, y \in \mathbb{R}$, there exists $t \in \mathbb{R}$ with $\varphi_t(x) = y$.